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Map-making methods for Cosmic Microwave Background experiments

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ABSTRACT

The map-making step of Cosmic Microwave Background data analysis involves linear inversion problems which cannot be performed by a brute force approach for the large timelines of today. We present in this article optimal vector-only map-making methods, which are an iterative COBE method, a Wiener direct filter and a Wiener iterative method. We apply these methods on diverse simulated data, and we show that they produce very well restored maps, by removing nearly completely the correlated noise which appears as intense stripes on the simply pixel-averaged maps. The COBE iterative method can be applied to any signals, assuming the stationarity of the noise in the timeline. The Wiener methods assume both the stationarity of the noise and the sky, which is the case for CMB-only data. We apply the methods to Galactic signals too, and test them on balloon-borne experiment strategies and on a satellite whole sky survey.

Key words: cosmology: Cosmic Microwave Background – methods: data analysis.

1 INTRODUCTION

The Cosmic Microwave Background (CMB hereafter) is being extensively studied nowadays, thanks to the improvement of instrument and detector performances. Since the COBE experiment (Smoot *et al.* 1992), which performed the first detection of CMB anisotropies, the size of time-ordered information (TOI hereafter) has been widely increased. The data processing and analysis is thus still a challenge for large timeline data, already in processing or still to come. MAP (see for example Wright 1999) and Planck (see for example Tauber 2000) experiments will produce huge timelines of hundreds of million samples or more, and even the balloon-borne experiments, BOOMERanG (De Bernardis *et al.* 2000), MAXIMA (Hanany *et al.* 2000), Archeops (Benoît *et al.* 2001) and many others, produce large timelines for many channels. The Cosmic Microwave Background data analysis is usually performed in three steps, the first being the map-making process, the second the C_l estimation from the maps and the noise covariance matrices, the third the cosmological parameters estimation from the C_l power spectrum. In this article, we focus on the map-making step, in the context of computing difficulties due to large timelines. We also explore how the scanning strategy influences the map-making process and the final amount of noise in the maps. Map-making, in the context of large timelines, cannot be performed by a brute-force approach, which would imply the manipulation and inversion of tera-element large matrices. Non optimal map-making methods have been developed, such as destriping using the scan intercepts (see Delabrouille 1998 and Dupac & Giard 2001). We aim in this paper to apply the optimal methods to large timelines, developing algorithms to avoid computation trouble. The map-making methods for CMB are known to be optimal when following desirable properties, such as minimizing the reconstruction error and the noise chi square towards the reconstructed map, and being the maximum likelihood solution for simple conditions or no conditions. These are linear map-making methods, well known as the COBE method (Janssen *et al.* 1992) and the Wiener filter (Wiener 1949). The linear methods allow to find an analytical expression for such requirements: the time-ordered data, represented by a vector y, are a linear expression of the real sky, represented by a vector x. (We note every vector with a small letter and every matrix with a capital letter.) We have thus:

$$y = Ax + n \quad (1)$$

where A is the point-spread (convolution) matrix and n the noise vector in the timeline. For CMB experiments, x would represent the pixelised sky map of the CMB temperature. The map-making problem is thus written as this:

$$\tilde{x} = Wy \quad (2)$$

where $\tilde{x}$ is the vector of the reconstructed sky map, and W the inversion matrix. The COBE method can be derived by minimizing the noise chi square towards the reconstructed map, the reconstruction error subject to the constraint W A = I, and is the maximum likelihood solution

for the sky if the noise is gaussian (no prior on the sky) (see for example Tegmark 1997). It is written as:

$W = [A^t N^{-1} A]^{-1} A^t N^{-1}$ (Eq. 3, COBE)

where N is the covariance matrix of the noise in the timeline: N $= < nn^t >$. The Wiener filter can be derived as the solution of the absolute minimization of the reconstruction error (Bunn *et al.* 1994), or as the maximum probability solution for the sky if both the noise and the sky are *a priori* gaussian (Zaroubi *et al.* 1995). It is written as:

$W = [S^{-1} + A^t N^{-1} A]^{-1} A^t N^{-1}$ (Eq. 4, Wiener 2)

or:

$W = S A^t [A S A^t + N]^{-1}$ (Eq. 5, Wiener 1)

where S is the covariance matrix of the sky in the map domain: S $= < xx^t >$. These two writings are the same matrix but can be totally different toward algorithmics. From these theoretically optimal methods, one can imagine other slightly different methods, made to solve such or such practical problem (see Tegmark 1997 for a large set of linear and non linear methods). The COBE method (Eq. 3) and the Wiener filter (Eq. 4, 5) are both known to retain all the cosmological information present in the TOI (Tegmark 1997). These methods are optimal subject to the condition that the noise is uncorrelated with the sky, which is a good assumption for low contrasts maps such as CMB ones. One finds this assumption in every paper concerning CMB map-making, and it is not the purpose of this article to question this, however, following the objects observed and the nature of the observation strategy, the bolometer response could be less perfect, and therefore induce some noise correlated to the sky signal.

Applying these optimal methods to large timelines is not straightforward, because of computer limitations. The several tens or hundreds of mega-elements in a bolometer timeline have to be processed by vector-only methods, that we aim to present in this paper. We mean by "vector-only methods" computations in which one never needs to compute or even to store any large matrix such as A, N or S. In this context, the way to make the timeline, that is the scanning strategy, can be important towards map-making and the quality of the final products. We present in Section 2 diverse balloon and satellite scanning strategies for CMB experiments. These aim to be representative of the different priorities one aims to focus on: redundancy, coverage, whole sky survey..., and how these strategies influence the map-making efficiency and the final quality of the retrieved maps. In Section 3 we present how we simulate our timelines. In Section 4 we present the map-making methods, applied to large timelines, and discuss their conditions of application. In Section 5 we compare the results on the reconstructed maps, for the different map-making methods and experiment strategies that we have investigated.

2 OBSERVATION STRATEGY

We present here three different large-coverage observation strategies, simulated from simple and usual technical requirements. We simulate balloon-borne CMB experiments, required to scan the sky making constant elevation circles at a rather constant rotation speed (as do Archeops and TopHat: http://topweb.gsfc.nasa.gov). This allows to observe a large area on the sky, thanks to the rotation of the Earth (and, eventually, to the moving of the balloon on the Earth). In this context, it is easy to understand that the launching place of the balloon and the flight duration will have a crucial influence on the sky coverage and the redundancy in the map pixels. Experiments using this constant elevation scanning strategy have to balance between coverage and redundancy. Maximum redundancy is obtained with high elevation angles and high latitude launch places. To the contrary, the best coverage over flight time ratio is obtained at near-equatorial places and with low elevation angles. The factors controlling the observation characteristics are the place and date of launch, the trajectory of the balloon, the flight duration, the elevation angle of scanning above the horizon, the sampling frequency, and the rotation speed of the gondola (especially if it is constant or not). We have simulated two balloon-borne experiment timelines with a constant scanning elevation of 35 degrees above the horizon, a sampling frequency of 100 Hz and a rotation speed of the gondola of 2 rpm.

Table 1. Caracteristics of the three observing strategies.

	Polar	Equatorial	Satellite
Observing time	24 h	12 h	(under-sampling)
Coverage	35 %	61 %	100 %
Number of samples (millions)	8	4	8
Avg redundancy per pixel	30.5	8.7	11.8

One simulated experiment is designed to maximize the redundancy per pixel, however covering a large part of the sky. It is a 24 hours polar flight from Kiruna, Swedish Lappland. The winter time in this region provides polar nights allowing to make 24 hours flight avoiding contamination from the sun. The coverage for this polar flight is 35 % of the sky.

The second simulated experiment is a 12 hours equatorial flight. The coverage is for this 12 hours flight 61 % of the whole sky, and would be 82 % for a 24 hours flight, or two 12 hours flights, one taking place six months after the first one, to avoid sunlight. The polar flight we present here is especially good for redundancy, while the equatorial flight has a clearly better coverage but a much weaker redundancy. The comparison of map-making methods on these two flights will therefore give interesting results on how in practice the observation strategy controls the final quality of the maps.

The third simulated scanning strategy is the one of a satellite making great circles on the sky along ecliptic meridians. For a satellite on a low-earth orbit (LEO), this strategy allows to always point towards the zenith, in order to avoid contamination from the Earth. The great circles are made perpendicularly to the sun direction, that allows to avoid the main sunlight contamination too. Of course, on a LEO, the exact trajectory would not be exactly as simple, because of the gravitational perturbations that the satellite would face, but our simulated timeline is near to what it would be. This strategy also works for a satellite at L2, as Planck. In this case the strategy can be as simple as a con-

Figure 1. Top: whole sky simulation of CMB fluctuations with $\Omega_\Lambda = 0.7$, $\Omega_{CDM} = 0.25$, $\Omega_{bar} = 0.05$, $H_0 = 50$. Bottom: whole sky simulation of CMB fluctuations and the Galaxy at 2 mm wavelength. The maps are Galactic Mollweide projections centered on the Galactic center.

Figure 2. Top: map of the polar flight data. Center: map of the equatorial flight data. Bottom: map of the satellite data. The three maps contain the CMB fluctuations, the dipole, the Galaxy and the instrumental noise.)

stant great circle in the ecliptic meridian perpendicular to the Sun-Earth axis, that slowly shifts with the revolution of the Earth. The whole sky survey is thus completed in six months. We have simulated a quick whole sky survey made with this observing strategy, containing 8 million samples. Although this would not be of course the final product of a one-year satellite mission, it allows us to compare the scanning strategy and final map properties with those of the two balloon-borne experiments.

3 SIMULATION PROCESS

We have made our simulated skies from three components: the CMB fluctuations, the dipole, and the Galaxy, at 2 mm wavelength. The simulated Universe we have simulated is Λ dominated, with $\Omega_\Lambda = 0.7$, $\Omega_{CDM} = 0.25$, $\Omega_{bar} = 0.05$, $H_0 = 50$ and a scalar spectral index of the fluctuations equal to 1. The exact cosmological parameters we choose is not important for the purpose of this article, but the gaussianity is *a priori* important for the Wiener methods that we will test. The C_l simulated spectrum (with no cosmic variance) is made thanks to the CMBFAST software (Seljak & Zaldarriaga 1996). The sky gaussian random field (i.e. a realization of a Universe with the cosmological parameters we chose) is then simulated with the SYNFAST tool of the HEALPix package (http://www.eso.org/science/healpix). The dipole is added thanks to its COBE/DMR determination (Lineweaver *et al.* 1996), and the Galaxy at 2 mm wavelength is extrapolated from the composite 100 μm IRAS-COBE/DIRBE all sky dataset (Schlegel *et al.* 1998). We present in Fig. 1 a full-sky map of the Cosmic Microwave Background fluctuations, and a full-sky 2 mm map including the dipole and the Galaxy. All the maps that we present in this article are Galactic Mollweide projections centered on the Galactic center.

We have simulated noisy timelines. However, the true time-ordered data coming directly from a true detector has not exactly the same shape. Usually there are glitches, systematics and noises to be subtracted before applying careful map-making methods. The glitches are energetic cosmic rays which hit a bolometer and produce high and narrow peaks in the time-ordered data. They can usually be detected and removed. Some noises and systematics may be removed by decorrelating with signals from thermometers or other bolometers. The topic of this article is not to treat these problems, however it is important to keep in mind that the map-making that we present here is not known to work well on brute data. For our simulations, we aim to test our map-making methods with white and 1/f noise. The noise we introduce in our simulated timelines can be characterized by its statistical power spectrum which follows a 1/f law: $l_{inf}.(1 + (f_c/f)^n)$, where l_{inf} is the level of white noise (i.e. the only noise at high frequency), f_c the cut frequency and n the power index. These stationary noises are usually what stays in the cleaned timelines, after removing what is possible to remove without modifying the signal. In order to properly compare our different observing strategies, we apply the map-making methods to timelines simulated with the same noise level and statistical properties, with $n = 1$, $f_c = 0.1$ Hz and $l_{inf} = 100\ \mu K_{CMB}$ rms. This white noise rms is approximatively the level expected for Planck (Tauber 2000) bolometers with a 100 Hz sampling rate. Of course this quite low value is the level of the non-correlated noise in the timelines, but the total amount of noise introduced is much larger, as we can see in Fig. 2, which is the three simulated data projected on a map by averaging the samples in the pixels (the simplest map-making method, as we explain hereafter).

4 MAP-MAKING METHODS APPLIED ON LARGE TIMELINES

To make maps, we need first to choose a pixelization of the sphere. We use the HEALPix scheme (http://www.eso.org/science/healpix), whose angular resolution is defined by the parameter N_{side}. We present our simulated and reconstructed maps with $N_{side} = 256$, which splits the sphere into 786432 pixels. This number of pixels corresponds to a 13.5' by 13.5' pixel on the sphere, and fits well the scanning strategies we simulate. Indeed, using $N_{side} = 512$ pixels would leave many holes in the maps and therefore harm their visibility and further analysis. To pixelize correctly the maps of a given experiment, and do the further analysis, one needs of course to use pixels two or three times smaller than the beam diameter. Maps of many modern experiments would be well pixelized with pixels a bit smaller than we use, but it would mean large maps to store and a bit longer computation times (as the convergence speeds of the methods would be lower), without giving any extra result to these investigations. In this article, we do not deal with the beam of an experiment and we do not treat the issue of deconvolving the beam effect.

4.1 Pixel averaging

Reprojecting timelines on maps is not only a domain to domain transform, but the simplest way to estimate the true map of the sky, by averaging the samples of a same pixel on the sky. The noise is therefore reduced by a factor square root of the number of samples in the pixel (the weight). To increase the signal-to-noise ratio in the pixels, one has to increase the counts in a pixel, so increase the pixel size. However the size pixel limits of course the maximum l to be investigated ($l_{max} = 2.N_{side}$). If we consider pixel sizes bigger than the beam, or neglect the beam effect, then the A^t matrix is adding samples in a pixel, which means that this matrix is filled with 1 and 0 only. This is much easier to

Figure 3. Weight maps: polar flight on top, equatorial flight in the center, satellite at bottom. The units are in number of timeline samples in the pixel.

handle than the true point-spread matrix, as in this simple case the map-making (pixel averaging) of a timeline y is just $[A^tA]^{-1}A^ty$, where A^tA is a hit counter per map pixel. A is in this case just the projection from a map to a timeline. We will not investigate the beam deconvolution in this article, and therefore consider only 1-and-0 point-spread matrices. We present in Fig. 3 the weight maps for the three experiment strategies that we have described. It is clear that the 24 h balloon-borne polar flight has the best redundancy. We will see if these differences in the experiment strategy produce important differences in the final reconstructed maps.

4.2 Correlation properties

The optimal map-making methods use the noise and sky covariance matrices N and S. These are impossible to invert or even to store for such large timelines. However, if the noise is stationary in the time domain, as it is the case for 1/f noise and white noise (usual bolometer noises), the noise correlation matrix in the time domain is circulant, i.e. multiplying a vector by this matrix is a convolution, which is filtering in the Fourier domain. The noise correlation matrix has to be a statistical average on the realizations of the nn^t matrix, which means that a fit has to be performed on the Fourier noise vector, to remove the gaussian fluctuations. The Fourier noise vector can be estimated from the data, since the 1/f tail is usually visible. We fit this noise power spectrum with the $1/f^n$ law expressed below. For the sky covariance matrix, the problem is different. The sky covariance matrix is stationary in the map domain, as far as the Cosmic Microwave Background is a gaussian random field. Thus the S matrix (in the map domain) is circulant for CMB maps. However, it is more complicated for the sky covariance matrix in the timeline. The sky covariance matrix in the time domain is stationary if two conditions are verified: the sky covariance matrix has to be stationary in the map domain, and the sampling on the sky has to be constant, that means that the scanning speed has to be constant if the sampling frequency is constant, which is usually the case. If the last condition is not verified, then the stationarity of the CMB signal is lost in time domain. However, the tolerance threshold to this condition has to be investigated. If we consider both the sky timeline and the noise timeline to be stationary, then multiplying by S^{-1} or N^{-1} matrices is a convolution, i.e. a vector multiplication in the Fourier domain.

4.3 Direct and iterative methods

We present here three optimal vector-only map-making methods: a COBE iterative method, a Wiener direct method and a Wiener iterative method.

4.3.1 COBE iterative method

The COBE equation (Eq. 3) cannot be directly applied with vector-only algorithms, because of the matrix inversions needed. Thus the trick is to solve rather:

$[A^tN^{-1}A]\ \tilde{\mathrm{x}} = A^tN^{-1}\ \mathrm{y}$

This form prevents from the heavy inversion, but needs an iterative scheme. The general iterative scheme for this equation is:

$\alpha\ \tilde{\mathrm{x}}_{n+1} = \alpha\ \tilde{\mathrm{x}}_n + A^tN^{-1}\ \mathrm{y}$ - $[A^tN^{-1}A]\ \tilde{\mathrm{x}}_n$

where α is any linear operator on a vector, that is, any square matrix. We have tested this algorithm on simulations and real data from the Archeops experiment (Benoît *et al.* 2001), with α being a scalar. By testing the method with different α, we find that the identity is the best iterator.

Another scheme can be developed, by making the noise map converge instead of the sky map, as mentioned by Prunet (2001). This can be better, as the signal can be more tricky than the instrumental noise for the stability of the iterative scheme: hot galactic points for example may induce stripes on the maps. The noise-iterating scheme works with the following trick: we change the variable $\tilde{\mathrm{x}}$ to $\check{\mathrm{x}} = [A^tA]^{-1}A^t$ y - $\tilde{\mathrm{x}}$. It is straightforward to show that this is the noise map plus the reconstruction error. It leads to:

$\alpha\ \check{\mathrm{x}}_{n+1} = \alpha\ \check{\mathrm{x}}_n + A^tN^{-1}\ \mathrm{z}$ - $[A^tN^{-1}A]\ \check{\mathrm{x}}_n$

where z = $A[A^tA]^{-1}A^t$ y - y. If this algorithm converges, then the converging limit is exactly the optimal solution of the map-making problem: the proof is that if it converges, then $\check{\mathrm{x}}_{n+1} = \check{\mathrm{x}}_n$ at the limit, and so the above equation leads to the map-making equation for $\check{\mathrm{x}}_n$, which is therefore the unique solution. Of course, this true solution is different from the true (simulated) sky, the residual noise covariance matrix being $[A^tN^{-1}A]^{-1}$ in the case of the COBE method. The case of α = I is actually the simplest iterator one can imagine, but works well on the simulations and real data that we have processed. We present in Section 5 the maps made with the iterative noise-converging COBE method.

4.3.2 Wiener direct method

The Wiener 1 matrix (Eq. 5) is reduced to the Wiener filter (Wiener 1949) in the time domain, if few conditions are verified. Indeed, the Wiener 1 equation can be written as:

$W = [A^tA]^{-1}A^tASA^t[ASA^t + N]^{-1}$

ASA^t is the sky covariance matrix in the timeline, and N is the noise covariance matrix in the timeline. Thus if both the noise and the sky are stationary in the time domain, then the Wiener 1 method is mapping the filtered timeline, with the optimal Wiener filter being $\frac{\sigma^2}{\sigma^2+\nu^2}$ in the Fourier domain, with σ and ν the fitted Fourier transforms of the sky and the noise. It is important to notice that two conditions are necessary so that the signal is stationary in the timeline: the signal has to be stationary in the map, and the observation has to be stationary towards the map, which means that both the scanning speed and the sampling frequency have to be constant. The noise Fourier fit is made as described for the COBE iterative method. The sky power spectrum in the timeline is dominated by the rotation peaks, and so to approximate the realization average by the power spectrum itself is not a bad approximation. This can be easily done even for real data by simulating the observation of a pure sky

Figure 4. Power spectra of the polar flight noise timeline (black) and the sky timeline (blue). The rotation peaks on the sky power spectrum are clearly visible.

Figure 5. Top: evolution of the difference map rms between the reconstructed sky and the true simulated sky. The level of white noise is shown as a blue straight line at 21.1 μK_{CMB}. Bottom: evolution of the maximum of the residual noise map.

(without noise). We present in Fig. 4 the power spectra of the simulated noise timeline and the simulated sky timeline.

The Wiener direct method is optimal when applied to stationary gaussian random signals, but we test it also on non-gaussian and non-stationary skies, the first being non-stationary even in the map domain because of the Galaxy, the other being non-stationary on the timeline because of large variations of the scanning speed, and we present the results in Section 5.

4.3.3 Wiener iterative method

The Wiener 2 matrix (Eq. 4), although identical to the Wiener 1, is computationally nearer to the COBE matrix (Eq. 3). Thus the iterative scheme is very close to the COBE one:

$\alpha\ \check{x}_{n+1} = \alpha\ \check{x}_n$ + u - $[S^{-1} + A^tN^{-1}A]\ \check{x}_n$

where u $= A^tN^{-1}[A[A^tA]^{-1}A^ty - y] + S^{-1}[A^tA]^{-1}A^ty$.

As we have shown for the COBE iterative method, the converging limit is the exact solution of the Wiener map-making equation. This iterative scheme needs to handle both the N matrix, noise covariance matrix in the timeline, that we process as a filter in the Fourier domain like we do for the COBE iterative method, and the S matrix, sky covariance matrix in the map domain. Handling this as a matrix is not possible for a small scale pixelization that we need for CMB experiments of today, thus we have to process it as a filter in Fourier space, like we do for filtering timelines. The HEALPix RING scheme is stationary with respect to the sphere, because it pixelizes it making a ring around the sphere from the north pole to the south pole, with equal pixel surfaces. So filtering a HEALPix vector (i.e. a map) in Fourier space is optimal, to the condition that there must not be large holes in the map, that would harm the stationarity of the sky in the HEALPix scheme. Of course this means that for maps like the polar flight ones or the equatorial flight ones, the filtering in the HEALPix scheme has to be made separatly for each observed part of the sky, following the HEALPix ordering. Another method is to define another pixelization, adapted to a given observation. But our satellite whole sky survey experiment is particularly well adapted to test this method, as the holes are few in the $N_{side} = 256$ pixelization, so we can just filter the whole map. We present the resulting maps in Section 5.

5 RESULTS AND DISCUSSION

5.1 Testing the COBE iterative method and the Wiener direct method on the polar flight data

We have made 1000 iterations with the COBE method, for the polar simulated data containing the CMB fluctuations, the dipole, the Galaxy and the instrumental noise. We present the convergence plot of the rms of the difference map between the reconstructed sky and the true sky (residual noise map) in Fig. 5 (top). The level of white noise is shown as a blue straight line at 21.1 μK_{CMB} rms. At the bottom of Fig. 5, we present the convergence plot made by considering at each iteration the less well reconstructed pixel of the map, i.e. the maximum of the residual noise map.

The method reaches the residual noise at about 50 iterations, and this residual is about 21.6 μK_{CMB} rms. We can check that we have reached the convergence by observing the evolution of the global residual noise rms, but also the evolution for some individual pixels, the map aspect and the C_l power spectrum. We have to compare this result to the white noise amount in the map. We can calculate the white noise in the map by two ways: one is to calculate it theoretically as $n_{rms}^2.\Sigma_i w_i^{-1}/N_{pix}$, where n_{rms} is the white noise rms level in the timeline, w_i the weight in pixel i, and N_{pix} the total number of observed pixels (the sum is of course on the observed pixels). The other is to simulate a white noise timeline and make a map. The rms level of white noise in the polar flight map is 21.06 μK_{CMB} rms, which is very close to the residual noise amount. This shows how good the reconstruction is, as it is clear that the correlated noise is significantly removed from the map. The reconstructed map exhibits no visible difference with the true map (whole simulated sky in Fig. 1). We present in Fig. 6 the residual noise map. The noise feature which can be seen in this map is the granularity of the white noise, which confirms that the iterative method removes nearly completely the correlated noise. Moreover, the bottom plot in Fig. 5 shows that there are no pixels excluded from the general convergence. We present in Fig. 7 the C_l power spectra of the map made by simple pixel-averaging, of the residual noise map after convergence, and of a map made with only white noise at the same level. All spectra are corrected from the fraction of the sky covered. The units show very well how better is the iterative method, compared to the pixel averaging. The noise spectrum is nearly the one of a white noise above about l=50, but exhibits some weak residual correlation at larger scales. Since we can be confident in the fact that the convergence has been reached, then the map obtained has to be the solution of the map-making problem. We see that a small amount of correlated noise still lurks at large scales: this may be explained by the fact that these scanning strategies do not constrain perfectly the largest scales (few crossing scans), which are the most noisy (by definition of the 1/f noise). Of course, a method such as MASTER (Hivon *et al.* 2001, subm. to *ApJ*) may be able to estimate this large-scale noise excess. However, this correlated residual noise amount is extremely low, even at the largest scales, and could have to be taken into account only for extremely precise measurements of the low l (precision better than 0.05 μK^2 on the C_l at large scales !).

We have also tested the method on timelines made with just CMB fluctuations and instrumental noise. To get a timeline without the Galaxy from real data, it is possible, for

Figure 6. Residual noise map obtained with the COBE iterative method applied on the polar flight data. The much largest part of this residual is clearly the averaged white noise.

Figure 7. Top: C_l power spectrum of the polar flight residual noise map made by pixel averaging. Bottom: C_l power spectra of the residual noise map after convergence (black), and of a white noise map (blue). All spectra are corrected from the fraction of the sky covered. The units show very well how better is the iterative method, compared to the pixel averaging. The noise spectrum is nearly the one of a white noise above about l=50, but exhibits some weak residual correlation at larger scales.

instance, to decorrelate with a shorter wavelength bolometer. Indeed, most CMB experiments have high frequency channels (for instance Planck and Archeops bands at 353 and 545 GHz) in which the CMB fluctuations are completely negligible. These bands are used as tracers of the dust Galactic emission. The way to decorrelate the CMB bands from the dust emission is usually to do it after the map-making process (component separation), but it is also possible to decorrelate the Galactic emission in the timelines, though it may cause noise propagation in the CMB-band timeline. The right way to do it seems to us (though it is not the purpose of the paper) to get first a Galactic map as clean as possible (from some high frequency bolometers, or even from standard Galactic maps as IRAS ones...), and then re-sample it with the scanning strategy of the timeline one wants to decorrelate. But we have to note also that many CMB experiments do not observe the Galaxy at all, and thus have directly nearly CMB-only timelines.

The residual noise obtained on the reconstructed CMB-only map is the same as the one obtained from complete data. This shows that the COBE iterative method is as efficient with different sky signals, as we could expect from the theory. This algorithm is very efficient, but quite slow: the full CPU time on a 500 MHz PC is about one hour per 25 iterations, for the 8 million sample timeline. However, we shall see hereafter that the number of iterations needed to reach the convergence can be much reduced, if we use a first estimator for the sky map. Also, it is possible to speed up this kind of iterative methods with multi-grid algorithms (Doré *et al.* 2001).

We have applied the Wiener 1 direct method to the same timeline. This method is particularly well adapted to CMB-only signals, as the stationarity is perfectly verified, and more fundamentaly because the Wiener method assumes a gaussian prior for both the sky and the noise. The direct Wiener method works very well on the CMB-only data: the difference map between the Wiener direct reconstructed map and the true sky is 22.8 μK_{CMB} in rms, which is close to the white noise level. One could wonder what a "wild" whitening filter used instead of the optimal Wiener filter could produce: we present in Fig. 8 the map made with a whitening filter, that is the inverse of the fitted noise power spectrum. The result is that the signal is clearly destroyed by this non-optimal filter (compare with the CMB map in Fig.1).

Table 2. This table presents the noise amounts in the maps made with data containing the CMB fluctuations and the instrumental noise. The first line shows the noise rms in the coadded maps, the second shows the rms of the white (uncorrelated) noise, the third shows the residual noise rms after convergence of the iterative COBE method, the fourth after the iterative Wiener method. The last line shows the residual noise rms after the Wiener direct filtering. The units are in μK_{CMB}, and n.c.s.s. stands for non constant scanning speed.

	Polar 24 h	Polar 24 h n.c.s.s.	Equat. 12 h	Satel.
Total noise rms in the map	349.7	352.5	498.6	378.4
White noise rms in the map	21.06	21.37	40.84	38.14
Residual rms COBE iter.	21.61	-	42.13	38.93
Residual rms Wiener iter.	-	-	-	38.93
Residual rms Wiener direct	22.82	33.82	32.34	41.40

Figure 8. CMB map constructed with a whitening filter, from the polar flight data. The signal is clearly destroyed by this non-optimal filter.

We present in Fig. 9 the residual noise map obtained with the Wiener direct method.

Some weak residual striping is still visible in this map. It shows that the Wiener direct filtering processes the data in a very different way than does the COBE iterative method. The iterative method removes gradually the correlated noise, to end with a nearly perfectly destriped map, whereas the Wiener direct filter processes the data in order to properly construct the sky map. In this method, it is the signal which is restored, and not the correlated noise which is removed. This may explain why some correlated noise can be in the final map while the total residual noise amount is very weak. It is important to keep in mind that not only the quantity of residual noise is important, but above all its "whitness": if the residual noise is quite correlated, then the pixel noise matrix in the reconstructed map is not diagonal, which is a problem for the C_l estimation. The C_l spectrum of the Wiener-direct-reconstructed map shows that some power is suppressed at all scales, whereas it is not the case for the COBE iterative method, for which the C_l power spectrum exhibits no significant difference with the true CMB power spectrum.

An idea to accelerate the convergence of the iterative

Figure 9. Residual noise map obtained with the Wiener direct method applied on the polar flight data. There is some weak residual striping, to the contrary to what is obtained after iterations.

methods is to use a non-zero first estimate of the map, the best being the Wiener direct map. It is important to understand that whatever is the first estimate, what defines the exact solution is only the map-making equation, thus the Wiener 1 first estimate used for COBE iterations accelerates the convergence of the COBE iterative method, but does not influence the final reconstructed map.

In order to test how the lack of stationarity in the signal can harm the Wiener direct filtering, we test the method on a timeline made with CMB and Galactic signals, plus the instrumental noise. We also test it on a CMB timeline suffering from an extremely non constant scanning speed, the beam doing 30 degrees amplitude swingings 17 times per minute. The Galactic Wiener direct map is quite badly reconstructed, since stripes are still visible and the residual noise is 257 μK_{CMB} in rms. This is not surprising, as the intense Galactic signal harms very clearly the stationarity of the sky signal. The Wiener direct filtering is thus not adapted to timelines containing Galactic signals. It may therefore be applied to timelines whose foregrounds have been removed first, or to observations made away from the Galactic plane. To the contrary, the Wiener direct map is well reconstructed for the data made with the very non constant scanning speed: the residual noise is 33.8 μK_{CMB} in rms, which is not bad compared to the 21.4 μK_{CMB} rms of white noise in the map, but of course clearly less good than the 22.8 μK_{CMB} rms of residual noise obtained with the same processing on constant scanning speed data. However, the good result obtained with these dreadfully sampled data shows the robustness of the method.

5.2 Comparing the different observations toward the map-making methods

We have applied the COBE iterative method with the Wiener 1 direct map as first estimate, to the equatorial flight and satellite data, with CMB fluctuations and instrumental noise. Once again, this can also be applied to signals including the Galaxy, but since the Wiener direct estimate would not be so good, the number of iterations needed to get a perfectly destriped map would be higher. The maps made with the COBE iterative method for these two other experiments are very well reconstructed too: for the equatorial flight simulations, the residual noise map is 42.1 μK_{CMB} in rms, with 40.8 μK_{CMB} rms of white noise. For the satellite data, the residual noise map is 38.9 μK_{CMB} rms, with 38.1 μK_{CMB} rms of white noise. The residuals of the iterative COBE method are close to the white noise amount, and the residual over white noise ratios are nearly constant. Of course the residual is larger for the low-redundancy experiments, but the efficiency of the iterative method is not less. In particular, this shows that numerous crossing scans are not crucial for the map-making process.

It is interesting to notice that in the case of the equatorial flight, the Wiener direct filtering gives a lower residual than the COBE iterative method, and even lower than the white noise amount. However, the residual noise map (Fig. 10) exhibits some residual striping.

Figure 10. Residual noise map obtained with the Wiener direct method applied on the equatorial flight data. There is some residual striping, even if the total residual noise amount is extremely weak.

Figure 11. Residual noise map obtained with the Wiener iterative method applied on the satellite data. The granularity of the white noise is clearly visible.

5.3 Testing the Wiener iterative method on the satellite simulated data

As explained before, a whole sky survey, even with a few holes in the map, is a perfect candidate to test the Wiener iterative method. The method works well even with no first estimate of the sky, but is much faster with a Wiener 1 first estimate. We present in Fig. 11 the residual noise map of the Wiener iterative method. This map is very close to the one of the COBE iterative method (no visible difference, and very close rms: 38.933 μK_{CMB} for the COBE iterative method, 38.931 μK_{CMB} for the Wiener iterative method). Once again, we observe that the correlated noise is nearly totally removed by the iterations. The Wiener method is known to minimize the reconstruction error: we check it, but the difference with the COBE method is very small. Moreover, the C_l power spectra of the true CMB sky and both reconstructed maps (COBE iterative and Wiener iterative) do not exhibit significant differences.

5.4 Generalization of the iterative map-making methods

The noise-converging iterative scheme can be generalized to balance the influence of the *a priori* signal correlation knowledge in the map-making process. This is done with the following map-making matrix:

$$W = [\epsilon S^{-1} + A^t N^{-1} A]^{-1} A^t N^{-1} \qquad \text{(Eq. 6, Saskatoon)}$$

It is the Saskatoon way to make maps (Tegmark *et al.* 1997 and Tegmark 1997), where the ϵ factor allows to choose the signal-to-noise ratio in the final map. Actually, the Wiener map-making gives generally less noisy maps than the COBE one, by reducing the power in the pixels unequally. This induces a given signal-to-noise ratio in the final map, which is optimal with respect to the minimization of the reconstruction error, in the case of the Wiener method. As the noise is more important at small scales, the Wiener map-making smoothes the map in an adequate way. However, it is interesting to assume a different signal-to-noise ratio, in order, for instance, to try to retrieve small scale features in the map. For $\epsilon = 0.5$, we obtain a residual noise map of 38.932 μK_{CMB} in rms, that we could expect. The method is unstable for $\epsilon > 1$.

We have seen that the difference exists much more between the iterative methods and the direct ones, than between COBE iterative and Wiener iterative. Indeed, the iterative methods are very precise, both toward the map aspect and the C_l spectrum, and do not seem to allow significant differences between Wiener and COBE, with this

quite good signal-to-noise ratio. Though, we can be confident in the reality of the Wiener iterative method (compared to the COBE one) because the epsilon factor is very sensitive (when superior to 1, the method diverges).

6 CONCLUSION

We have presented efficient map-making methods for large Cosmic Microwave Background timelines. The direct way to estimate the sky signal from a noisy timeline is the Wiener direct filtering, which is robust towards scanning strategy non-stationary effects, but needs to process a stationary signal, which demand to remove the Galactic signal before map-making. The iterative methods (COBE, Wiener, and generalized) are very efficient to remove the correlated stationary noises, and allow to get a very good precision on the reconstructed map, whose residual noise is nearly only averaged white noise. Though the results of COBE, Wiener and "Saskatoon" iterative methods are very close, we check that the Wiener estimation more reduces the global reconstruction error. The difference should be larger if the pixels are more unequally noisy: it seems that this kind of experiment strategies does not allow the Wiener estimation to be very different from the *no prior* one. The different scanning strategies that we have simulated allow as efficient map-making, even if the crossing scans are few. However, it is important to keep in mind that we have processed stationary noises, which means a timeline free from, for instance, periodic systematics which could be projected on the maps like features on the sky.

This kind of vector-only methods seems to us unavoidable to make optimal maps from CMB experiments of today, or still to come. The reduction of the information in CMB data is a heavy work, from gigabytes of rawdata to essentially 12 cosmological numbers with their error bars. Since the computer facilities are limited and unsufficient for brute force approaches (and it will be still the case for Planck data reduction), it is an interesting challenge to process each step of this reduction work without losing information. Using stationarity properties of a signal in a given domain (sphere, map, timeline...) to transform a matrix inversion problem into a vector-only solution, could be probably also developed for other CMB reduction steps, such as the component separation.

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